

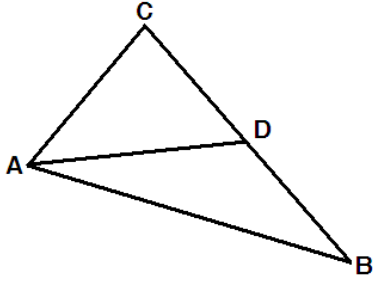
56th ANNUAL MATHEMATICS CONTEST 2010
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INSTRUCTIONS: Mark on the answer sheet the letter corresponding to the correct answer. Your score will be determined as follows: 4 (number of correct responses) – (number of incorrect responses). Answers left blank do not count either way. Do not guess wildly. USE A NUMBER TWO PENCIL TO MARK YOUR ANSWER SHEET. TIME OF THE CONTEST: 90 minutes. Total points 320.

- 1) Compute: $3 + 2 - (7 - 15)8 + (26 \div 2)2^2 =$
 a) 121 b) 81 c) 7 d) 166 e) -7
- 2) Solve for x: $5x^2 - 3x - 3 = 3$
 a) $\frac{3 \pm \sqrt{39}}{10}$ b) $\frac{-3 \pm \sqrt{37}}{10}$ c) $\frac{3 \pm \sqrt{129}}{10}$ d) 1, -1 e) No Real Solution
- 3) Two values of the linear function $g(x)$ are $g(2) = -7$ and $g(5) = 17$ what is the slope of the graph of $g(x)$?
 a) $\frac{-1}{8}$ b) $\frac{3}{10}$ c) 3 d) $\frac{10}{3}$ e) 8
- 4) If $x \bmod y$ is the remainder when x is divided by y , then $(65 \bmod 6) - (30 \bmod 5) =$
 a) 2 b) 3 c) 4 d) 5 e) 6
- 5) Solve for the variable x : $2x + 5 \geq -3$ or $3x - 4 < 5$
 a) $-4 \leq x < 3$ b) $x \leq -4$ or $x > 3$ c) $x < -4$ or $x \geq 3$
 d) All Real Numbers e) No Real Solutions
- 6) How far does a wheel of radius 2 feet roll along the level ground in making 141 revolutions?
 a) 141π ft. b) 282π ft. c) 564π ft. d) 846π ft. e) 1128π ft.
- 7) Find the equation in slope-intercept form of the line through the point $(2, -3)$ and perpendicular to $-7x - 8y = -38$
 a) $y = -4x + \frac{19}{4}$ b) $y = \frac{-1}{4}x - \frac{19}{4}$ c) $y = \frac{-8}{7}x - \frac{37}{7}$
 d) $y = \frac{8}{7}x - \frac{37}{7}$ e) $y = \frac{7}{8}x + \frac{7}{8}$
- 8) Find the numerical value of the expression when $x = -3$; $\frac{5 - 4x}{7x^2 + 7x + 3}$
 a) $\frac{-7}{45}$ b) $\frac{17}{87}$ c) $\frac{-7}{87}$ d) $\frac{17}{45}$ e) $\frac{-17}{81}$
- 9) Combine the fractions and simplify: $\frac{x}{x-2} + \frac{4}{x+2} - \frac{8}{x^2-4}$
 a) $\frac{x-8}{x-2}$ b) $\frac{x-8}{x^2-4}$ c) $6x+4$ d) $\frac{x+8}{x+2}$ e) 1
- 10) Find the value of "c" that completes the square: $y^2 - 17y + c$
 a) $\frac{-289}{4}$ b) $\frac{289}{4}$ c) $\frac{-81}{4}$ d) $\frac{81}{4}$ e) $\frac{-9}{2}$
- 11) The number 10101 in binary is equal to what value in base ten?
 a) 15 b) 17 c) 19 d) 21 e) 23

- 12) Find the missing value so that the line through the points has the given slope:
 $(-3, y)$ and $(-7, -20)$ slope = $\frac{31}{4}$
 a) -2 b) 11 c) 14 d) 15 e) 16
- 13) Simplify and write the product in scientific notation: $(7.4 \cdot 10^{-6})(6.75 \cdot 10^{-2})$
 a) $(4.995 \cdot 10^{-4})$ b) $(4.995 \cdot 10^{-7})$ c) $(4.995 \cdot 10^4)$ d) $(4.995 \cdot 10^{-8})$ e) $(4.995 \cdot 10^3)$
- 14) State the SUM of the solution(s) for the equation: $\frac{3m+5}{m-2} + \frac{1}{m} = \frac{7m^2+35m+28}{m^2-2m}$
 a) $-7\frac{1}{4}$ b) -6 c) -4 d) 0 e) 8
- 15) Find the solutions for the equation: $-k + \sqrt{30-5k} = -6$
 a) $\{6\}$ b) $\{1\}$ c) $\{6,1\}$ d) $\{6,-1\}$ e) No Real Solution
- 16) Solve the equation: $-|6-2x|+7 = -1$
 a) $\{1,0\}$ b) $\left\{\frac{-8}{3}, 0\right\}$ c) $\{1\}$ d) $\{-1,7\}$ e) No Real Solution
- 17) Given the midpoint and one endpoint of a line segment, find the other endpoint.
 Given: Endpoint: $(1,8)$ midpoint: $(-10,6)$
 a) $(-1,10)$ b) $(12,10)$ c) $(-21,4)$ d) $\left(-4\frac{1}{2}, 7\right)$ e) $(9,-3)$
- 18) If a spring stretches 0.9 m when a 6-kg weight is attached to it, how much will it stretch when a 8-kg weight is attached to it?
 a) 0.2 m b) 4.2m c) 1.2 m d) 3.2m e) 5.4 m
- 19) The measure of $\angle A$ is four times the measure of its complement, $\angle B$. Then $2 \cdot m\angle A + 3 \cdot m\angle B$ is:
 a) 180 degrees b) 190 degrees c) 196 degrees d) 198 degrees e) 252 degrees
- 20) If the hypotenuse of a right triangle has a length 7 meters and the sum of the lengths of the legs is 8 meters, then the area of the triangle is:
 a) $\frac{10}{3}m^2$ b) $3.5m^2$ c) $4m^2$ d) $3.75m^2$ e) $7.5m^2$
- 21) Find the length of a secant of the larger of two concentric circles with radii 10 cm. and 6 cm. if the secant is tangent to the smaller circle.
 a) 8 cm b) 12 cm c) 16 cm d) 12π cm e) 16π cm
- 22) Find the area of the quadrilateral with vertices of: $(0,0)$, $(3,1)$, $(1,3)$ and $(7,5)$
 a) 12 b) $\frac{35}{2}$ c) 29 d) $\frac{67}{2}$ e) 35

- 23) In triangle ABC, $\overline{AC} = \overline{CD}$ and $m\angle CAB - m\angle ABC = 40^\circ$. What is $m\angle BAD$?



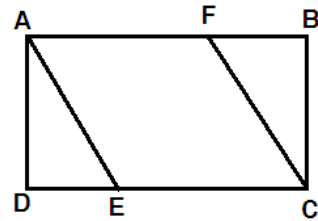
- a) 30 degrees b) 22 degrees
c) 20 degrees d) 15 degrees
e) 10 degrees

- 24) Find the area of the largest triangle that can be inscribed in a semi-circle whose radius is r ?

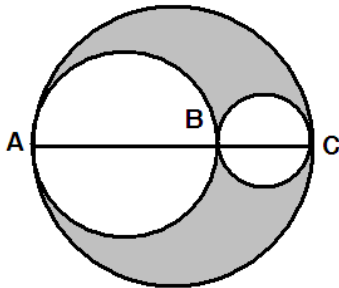
- a) r^3 b) $2r^3$ c) r^2 d) $2r^2$ e) $\frac{r^2}{2}$

- 25) On side \overline{AB} and \overline{DC} of the rectangle ABCD, points E and F are chosen so that AFCE is a rhombus. If $AB = 16$ and $BC = 12$, what is the length of AF?

- a) 15 b) 9
c) $\frac{7}{2}$ d) 12
e) $\frac{25}{2}$



- 26) In the figure shown, AC has a length 7, the semicircle \widehat{AB} has a radius of 2, semicircle \widehat{BC} has a diameter of 3. What percent of the big circle is shaded (round to the nearest units)?



- a) 24 % b) 33 %
c) 40 % d) 49 %
e) 56 %

- 27) Points A, B, and C lie on a circle and form an equilateral triangle. If $AB = 12$ what is the circumference of the circle.

- a) $\sqrt{3}\pi$ b) $4\sqrt{3}\pi$ c) $8\sqrt{3}\pi$ d) $10\sqrt{3}\pi$ e) $12\sqrt{3}\pi$

- 28) From 2:30 pm to 2:50 pm, how many degrees does the hour hand cross?

- a) 5 degrees b) 6 degrees c) 8 degrees d) 10 degrees e) 15 degrees

- 29) A circular grass plot 12 feet in diameter is cut by a straight gravel path 3 feet wide, one edge of which passes through the center of the plot. The number of square feet in the remaining grass area is:

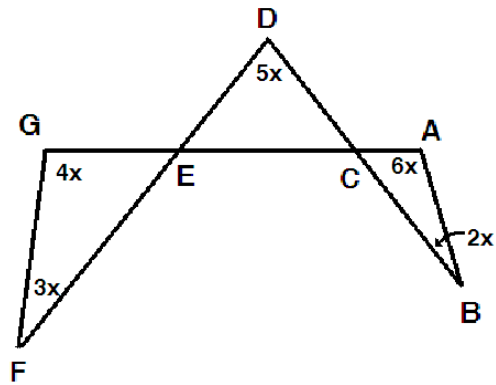
- a) $36\pi - 34$ b) $30\pi - 15$ c) $36\pi - 33$ d) $35\pi - 9\sqrt{3}$ e) $30\pi - 9\sqrt{3}$

- 30) The coordinates of triangle ABC are $A(2a, 2b)$, $B(2c, 2d)$, and $C(0, 2e)$. Find the slope of the altitude from B to \overline{AC} .

- a) $\frac{e-b}{a}$ b) $\frac{a}{e-b}$ c) $\frac{b-e}{a}$ d) $\frac{a}{b-e}$ e) $\frac{2a}{e-b}$

- 31) In the figure shown, what is the value of $\angle CAB$ In the diagram x is in degree measure.

- a) 54° b) 108°
 c) 120° d) 144°
 e) 162°

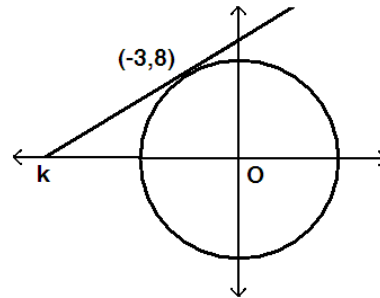


- 32) A square is inscribed inside a circle. The area of the square is what percent of the area of the circle (to the nearest tenth of a percent)?

- a) 63.7 % b) 31.8 % c) 15.9 % d) 40.6 % e) 50 %

- 33) In the figure shown, a line is tangent to the circle centered at the origin. The point of tangency is $(-3,8)$. The line intersects the x-axis at $x = k$. Find k.

- a) $\frac{-82}{3}$ b) $\frac{-73}{3}$
 c) $\frac{-64}{3}$ d) $\frac{-32}{3}$
 e) $\frac{-55}{3}$



- 34) A new homeowner has a triangular-shaped back yard. Two of the three sides measure 65 ft and 80 ft and form an included angle of 125 degrees. The owner wants to approximate the area of the yard. Find the area of the yard rounded to the nearest square foot.

- a) 1491 sq ft b) 2130 sq ft c) 2600 sq ft d) 4260 sq ft e) 5200 sq ft

- 35) A ship sailing parallel to the shore sights a lighthouse at an angle of 13 degrees from its direction of travel. After traveling 4 miles further, the angle is 23 degrees. At that time, how far is the ship from the lighthouse?

- a) 2.3 mi b) 3.1 mi c) 4 mi d) 5.2 mi e) 9.0 mi

- 36) A radio transmission tower is 140 feet tall. How long should the guy wire be if it is to be attached 9 feet from the top and is to make an angle of 28 degrees with the ground? (round to nearest tenth of a foot).

- a) 148.4 ft b) 158.6 ft c) 246.4 ft d) 279.0 ft e) 298.2 ft

- 37) Find the slope of a line perpendicular to the line containing the points $(2, -6)$ and $(-3, -8)$

- a) $\frac{5}{2}$ b) $\frac{-5}{2}$ c) $\frac{2}{5}$ d) $\frac{-2}{5}$ e) $\frac{-1}{2}$

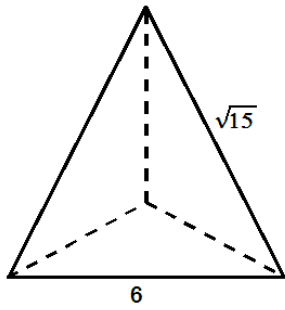
- 38) From the edge of a 1000-foot cliff, the angles of depression to two cars in the valley below are 21 degrees and 28 degrees. How far apart the cars (round to the nearest 0.1 ft.)?

- a) 147.8 ft. b) 383.9 ft. c) 654.7 ft. d) 714.4 ft. e) 724.4 ft.

- 39) Two points A and B are on opposite sides of a building. A surveyor selects a third point C to place a transit. Point C is 47 feet from Point A and 66 feet from Point B. The angle ACB is 52 degrees. How far apart are the points A and B (to the nearest tenth of a foot)?

- a) 101.9 ft b) 92.1 ft. c) 87.0 ft. d) 68.2 ft. e) 52.4 ft.

- 40) The volume of a pyramid whose base is an equilateral triangle of side length of 6 and whose other edges are each of length of $\sqrt{15}$ is:



- a) 9 b) $\frac{9}{2}$ c) $\frac{27}{2}$
- d) $9\frac{\sqrt{3}}{2}$ e) $9\frac{\sqrt{5}}{2}$
- 41) Find the volume of a right prism with a height of 6.3 feet and a base that is a parallelogram with a base of 4.2 feet and a height 3.7 feet.
a) 15.543 ft^3 b) 26.466 ft^3 c) 33.18 ft^3 d) 97.902 ft^3 e) 99.54 ft^3
- 42) A certain cylindrical pencil is 20 cm long, and has a diameter of 1 cm. The diameter of the lead is .5 cm. What is the volume, in cubic centimeters, of the wood of the pencil?
a) $\frac{15}{4}\pi$ b) $\frac{15}{2}\pi$ c) 5π d) 15π e) 20π
- 43) The *Golden Ratio* is defined as $\alpha = \frac{1+\sqrt{5}}{2}$. Evaluate $\alpha - \alpha^{-1}$
a) $-\left(\frac{3}{4} + \frac{\sqrt{5}}{10}\right)$ b) $\frac{5+\sqrt{5}}{1+\sqrt{5}}$ c) -1 d) 0 e) 1
- 44) How long will it take for an investment to triple if it earns 9.75% compounded continuously?
a) 11.27 years b) 5.63 years c) 11.93 years d) 7.20 years e) 10.82 years
- 45) The graph of the function $y = ax$, where $a > 0$, bisects an angle made by the graphs of $y = \frac{1}{3}x$ and $y = -3x$. Find the value of a .
a) 0 b) 1 c) 2 d) 3 e) 4
- 46) The solution of the inequality $\sqrt[3]{2+x+x^2} \leq 2$
a) 0 b) $-3, 2$ c) $x \geq -3$ d) $2 \leq x$ e) $-3 \leq x \leq 2$
- 47) Suppose the vertex of the curve of $y = (a+x)^2 + 2(a+1)x + 6$ is $(3, 1)$. Find the value of a .
a) 0 b) -1 c) -2 d) -3 e) -4
- 48) Suppose $f(x) = 2x+1$ and $g(x) = x-3$. Find the function $h(x)$ such that $f \circ h = g$
a) $2x - \frac{1}{2}$ b) $2x - 1$ c) $\frac{1}{2}x - 2$ d) $\frac{1}{2}x + 1$ e) $x - 4$
- 49) Find the product of the solutions to the equation: $2x^3 + x^2 - 13x + 6 = 0$
a) -6 b) -3 c) 3 d) 6 e) 12
- 50) Find the product of the solutions of the equation: $\ln(x^2) = (\ln(x))^2$
a) 2 b) e c) π d) e^2 e) no real solution(s)

- 61) The operation # is defined by: $x \# y = \frac{x^2 + y^2}{x + y}$ If $(x \# x) \# 3 = 5$ then which of the following could be a value for x ?
- a) -3 b) -2 c) 1 d) 4 e) 6
- 62) Solve the inequality: $\left| \frac{9y + 36}{4} \right| > 9$
- a) $(-8, 8)$ b) $(-8, 0)$ c) $(0, \infty)$ d) $(-\infty, -8) \cup (0, \infty)$ e) $(-\infty, -8) \cup (8, \infty)$
- 63) Solve for x : $2|x - 5| = |x + 5|$ Which of the following is the sum of the solution(s)?
- a) -20 b) $-\frac{50}{3}$ c) 15 d) $\frac{50}{3}$ e) 20
- 64) Find the slope of the line tangent to the curve at the given point. $y = 15 \sin(x)$; $x = \frac{\pi}{3}$
- a) $-\frac{15\sqrt{3}}{2}$ b) $-\frac{15}{2}$ c) $\frac{1}{2}$ d) $\frac{15}{2}$ e) $\frac{15\sqrt{3}}{2}$
- 65) Find the indefinite integral: $\int 9x^4 \cos(x^5) dx$
- a) $\frac{9}{5} \sin(x^5) + C$ b) $\frac{9}{5} x^5 \sin(x^5) + C$ c) $9 \sin(x^5) + C$
d) $\frac{1}{5} \sin(x^5) + C$ e) $45 \sin(x^5) + C$
- 66) Find the values of a , b and c such that the graph of the quadratic equation $y = ax^2 + bx + c$ passes through the points $(-2, 3)$, $(-1, 4)$ and $(4, 39)$
- a) $a = -1, b = 4, c = 7$ b) $a = 1, b = 7, c = 4$ c) $a = 1, b = 4, c = 7$
d) $a = -1, b = 7, c = 4$ e) $a = 1, b = -4, c = 7$
- 67) Find the volume of the solid generated by revolving the region in the first quadrant bounded by the graphs $y = x^2$, $y = 4$, $x = 0$ about the x -axis.
- a) $\frac{8}{3} \pi$ b) $\frac{16}{3} \pi$ c) $\frac{128}{5} \pi$ d) $\frac{32}{5} \pi$ e) $\frac{1024}{5} \pi$
- 68) Find the limit: $\lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x} - 4x + 6}{-2x + x^{2/3} - 5}$
- a) $-\infty$ b) $-\frac{1}{2}$ c) 0 d) $\frac{1}{2}$ e) 2
- 69) The centers of two circles (each with a radius r) are $3r$ units apart. A line is tangent to both circles and passes between them. What is the distance between the two points of tangency?
- a) $\sqrt{2} r$ b) $\frac{3}{2} r$ c) $2r$ d) $\sqrt{5} r$ e) $3r$
- 70) A container, in the shape of an inverted right circular cone, has a radius of 7 inches at the top and a height of 8 inches. At the instant when water in the container is 5 inches deep, the surface level is falling at a rate of -1.3 in/s. Find the rate at which the water is being drained.
- a) $-102.74 \text{ in}^3/\text{s}$ b) $-78.17 \text{ in}^3/\text{s}$ c) $-74.65 \text{ in}^3/\text{s}$ d) $-65.51 \text{ in}^3/\text{s}$ e) $-31.27 \text{ in}^3/\text{s}$

- 71) The n th term of a sequence is given. Calculate the fifth partial sum. $a_n = \frac{1}{n-8}$
- a) $\frac{-153}{140}$ b) $\frac{-223}{140}$ c) $\frac{-685}{252}$ d) $\frac{671}{1260}$ e) $\frac{2719}{1260}$
- 72) The position of a particle moving along the coordinate line is $s = \sqrt{3+6t}$ with s in meters and t in seconds. Find the particle's acceleration at $t = 1$ second.
- a) $1 \frac{m}{\text{sec}^2}$ b) $\frac{-1}{3} \frac{m}{\text{sec}^2}$ c) $\frac{1}{3} \frac{m}{\text{sec}^2}$ d) $\frac{-1}{18} \frac{m}{\text{sec}^2}$ e) $\frac{-1}{27} \frac{m}{\text{sec}^2}$
- 73) A man on a dock is pulling in a rope attached to a rowboat at the rate of 4 feet per second. If the man's hands are 3 feet higher than the point where the rope is attached to the boat, how fast is the angle of depression changing when there are still "d" feet of rope out? Give your answer in radians per second in terms of "d".
- a) $\frac{3}{d\sqrt{d^2-9}} \text{ rads/sec}$ b) $\frac{12}{d\sqrt{d^2-9}} \text{ rads/sec}$ c) $\frac{12}{d^2+9} \text{ rads/sec}$
d) $\frac{3}{\sqrt{d^2-9}} \text{ rads/sec}$ e) $\frac{6}{d(d-3)} \text{ rads/sec}$
- 74) A fox, initially at rest, with a position $s(0) = 0$, moves along a straight hunting path. At any time $t \geq 0$, its acceleration is $a(t) = e^{t/2}$, with $v(0) = 6$. How far does the fox travel from $t = 0$ to $t = 4$?
- a) 12 b) $3e^2 + 9$ c) $4e^2 + 12$ d) $4e^2 + 20$ e) 24
- 75) Find the area of the region enclosed by $16x^2 + 9y^2 = 144$
- a) 12 b) 12π c) 15π d) 16π e) 18π
- 76) Write the equation for the amount Q of a radioactive substance with a half-life of 11 days, if 7 grams are present when $t = 0$.
- a) $Q(t) = 7e^{-0.0630t}$ b) $Q(t) = 7e^{5.5t}$ c) $Q(t) = 7e^{-7.6246t}$
d) $Q(t) = 7e^{38.5t}$ e) $Q(t) = 7e^{-0.6931t}$
- 77) A curve is represented parametrically by $x = 2t^3$ and $y = 5t^4$. Find $\frac{dy}{dx}$
- a) $70t^6$ b) $\frac{5}{2}$ c) $\frac{10}{3}$ d) $\frac{5}{6t^2}$ e) $\frac{10t}{3}$
- 78) Given $y = \ln(x)$. What is the length of the arc of this curve over the interval $[1, 2]$?
- a) 1.015 b) 1.222 c) 2.222 d) 3.235 e) 4.128
- 79) Given: $\frac{24}{(x-4)(x+1)(x+2)} = \frac{A}{x-4} + \frac{B}{x+1} + \frac{C}{x+2}$. Decompose the given rational expression to find A, B, C .
- a) $A = -0.8, B = 4.8, C = -4$ b) $A = -0.8, B = -4.8, C = -4$
c) $A = 1.25, B = 0.8, C = 4.8$ d) $A = 0.8, B = 4.8, C = -4$
e) $A = 0.8, B = -4.8, C = 4$
- 80) $\frac{d}{dx} \int_2^{x^5} \frac{dt}{t+6}$
- a) $\frac{1}{x+6}$ b) $\frac{5x^4}{x+7}$ c) $\frac{5x^4}{x^5+6}$ d) $\frac{1}{x^5+6}$ e) $\frac{x^5}{x^5+6}$

Here are the answers.

1.	A	2.	C	3.	E	4.	D
5.	D	6.	C	7.	D	8.	D
9.	D	10.	B	11.	D	12.	B
13.	B	14.	A	15.	A	16.	D
17.	C	18.	C	19.	D	20.	D
21.	C	22.	A	23.	C	24.	C
25.	E	26.	D	27.	C	28.	D
29.	E	30.	B	31.	B	32.	A
33.	B	34.	B	35.	D	36.	D
37.	B	38.	E	39.	E	40.	A
41.	D	42.	D	43.	E	44.	A
45.	C	46.	E	47.	C	48.	C
49.	B	50.	D	51.	A	52.	B
53.	B	54.	C	55.	E	56.	C
57.	B	58.	A	59.	A	60.	A
61.	E	62.	D	63.	D	64.	D
65.	A	66.	C	67.	C	68.	E
69.	D	70.	B	71.	A	72.	B
73.	B	74.	C	75.	B	76.	A
77.	E	78.	B	79.	E	80.	C

Question # 76 in the original printing had a typo in answer "A" – Question omitted from scoring – Question in this copy is correct.