

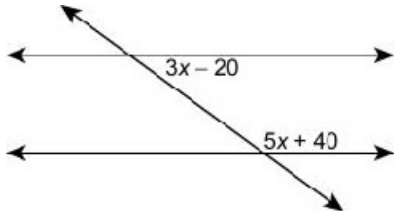
55th ANNUAL MATHEMATICS CONTEST 2009
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INSTRUCTIONS: Mark on the answer sheet the letter corresponding to the correct answer. Your score will be determined as follows: 4 (number of correct responses) – (number of incorrect responses). Answers left blank do not count either way. Do not guess wildly. USE A NUMBER TWO PENCIL TO MARK YOUR ANSWER SHEET. TIME OF THE CONTEST: 90 minutes. Total points 320.

- 1) Evaluate: $[(60 \div 6) \div 5 - 6] + 28 \div 4$
a) -6 b) 4.5 c) -3 d) 3 e) $\frac{19}{3}$
- 2) Solve for x: $3x^2 - 8x + 3 = 0$
a) $\frac{4 \pm 2\sqrt{7}}{3}$ b) $\frac{-4 \pm \sqrt{7}}{3}$ c) $\frac{4 \pm \sqrt{7}}{3}$ d) $3, \frac{-1}{3}$ e) $-3, \frac{1}{3}$
- 3) On a plot of land 18 feet by 30 feet a person adds 6 inches of topsoil. How many cubic yards were added?
a) 10 b) 30 c) 50 d) 120 e) 360
- 4) Some pitchers can throw a baseball as fast as 90 miles/hour. Approximately how many feet per second is this?
a) 116 ft/sec b) 120 ft/sec c) 122 ft/sec d) 128 ft/sec e) 132 ft/sec
- 5) A random sample of frogs is taken in a section of rain forest in Brazil. The sample contains 1 red frog, 6 blue frogs, and 45 green frogs. Researchers believe that an acre of this rainforest can sustain 25,000 frogs. Assuming that researchers are correct, use the ratios obtained from the sample to find how many blue frogs are in an acre?
a) 481 b) 1542 c) 2885 d) 4167 e) 7168
- 6) One thousand unit cubes are fastened together to form a large cube with edge length of 10 units. This is painted and then separated into the original cubes. The number of these unit cubes which have at least one face painted is
a) 600 b) 520 c) 488 d) 480 e) 400
- 7) The ratio of boys to girls in a certain class is $\frac{2}{3}$. If 4 more boys and 4 more girls are added to the class, the ratio of boys to girls will be $\frac{5}{7}$. How many boys are in the original class?
a) 16 b) 20 c) 24 d) 28 e) 32
- 8) The line $y = mx + b$ passes through (3,7) and (-4,-14). Find the sum of m and b.
a) $6\frac{1}{3}$ b) -7 c) 119 d) 19 e) 1
- 9) Find the quadratic whose solutions are $x = -6$ and $x = \frac{3}{4}$
a) $4x^2 + 27x - 18 = 0$ b) $3x^2 - 14x - 24 = 0$ c) $x^2 - \frac{9}{2} = 0$
d) $4x^2 + 21x - 18 = 0$ e) $3x^2 - 22x - 24 = 0$

- 10) How many revolutions per minute does a wheel make if its angular velocity is 120π radians per hour?
 a) 1 rpm b) 2π rpm c) $\frac{1}{60}$ rpm d) 3600 rpm e) 7200 rpm
- 11) Define the operation “@”, on real numbers as $a @ b = \frac{a}{2} + b$ Evaluate: $[x@(x-3)]@(x+5)$
 a) $3x+2$ b) $x^2+5x-15$ c) $\frac{5x+2}{4}$ d) $\frac{7x+14}{4}$ e) $x^3+2x-15$
- 12) If the graphs of $6x - 2y + 9 = 0$ and $ax + 4y - 6 = 0$ are perpendicular, then the value of “a” is?
 a) -7 b) $\frac{1}{3}$ c) $\frac{3}{4}$ d) $\frac{4}{3}$ e) 3
- 13) The number 234 in base five is equal to what value in base ten?
 a) 29 b) 39 c) 69 d) 234 e) 1170
- 14) Add and simplify: $\frac{2}{y^2-3y+2} + \frac{7}{y^2-1}$
 a) $\frac{12y-9}{(y-1)(y+1)(y-2)}$ b) $\frac{9y-12}{(y-1)(y+1)(y-2)}$ c) $\frac{28y-12}{(y-1)(y+1)(y-2)}$
 d) $\frac{9y-12}{(y-1)(y-2)}$ e) $\frac{12y-9}{(y+1)(y-2)}$
- 15) Find the equation of the line parallel to the line $5x+3y=4$ and has the x-intercept of -2 .
 a) $5x+3y=-2$ b) $3x-5y=10$ c) $5x+3y=-6$ d) $3x-5y=-6$ e) $5x+3y=-10$
- 16) How many integers are in the solution set of: $|4x+2| < 18$
 a) 6 b) 7 c) 8 d) 9 e) 10
- 17) A ladder is resting against a wall. The top of the ladder touches the wall at 9 feet high. Find the length of the ladder if the length is three more feet than the distance of the bottom of the ladder from the wall.
 a) 9 feet b) 12 feet c) 15 feet d) 18 feet e) 21 feet
- 18) Jill is 22.5 kilometers away from Joey. Both begin to walk towards each other at the same time. Jill walks at 1.5 km/hr. They meet in five hours. How fast is Joey walking?
 a) 1.5 k/hr b) 2.25 km/hr c) 3 km/hr d) 6 km/hr e) 15 km/hr
- 19) Solve the quadratic inequality: $x^2 - 4x > 12$
 a) $(-2,6)$ b) $(-\infty,-2) \cup (6,\infty)$ c) $(-\infty,-6) \cup (2,\infty)$ d) $(-6,-2)$ e) $(-\infty,\infty)$
- 20) If half of a number is added to five, the result is greater than or equal to a negative three. Find all such numbers.
 a) $x \geq -16$ b) $x \geq 2$ c) $x \leq -15$ d) $x \geq 4$ e) $x \geq -4$

- 21) In the figure, find the value of x with the angles given: $3x - 20$ and $5x + 40$



- a) 10 b) 15
c) 20 d) 25
e) 30

- 22) Find the equation of the perpendicular bisector of the segment joining the points $(-1, 1)$ and $(3, -5)$

- a) $2x - 3y = 4$ b) $2x + 3y = 4$ c) $2x - 3y = 8$
d) $2x + 3y = 8$ e) none of these

- 23) In a triangle, the measures of the angles are: $(x - 25)^\circ$, $(3x - 15)^\circ$, and $\left(\frac{2}{5}x\right)^\circ$. What is the measure of the smallest angle?

- a) 20 degrees b) 25 degrees c) 35 degrees, d) 40 degrees e) 50 degrees

- 24) An isosceles triangle has its shortest side half the length of the other two sides. The perimeter is equal to the area. Determine the length of the shortest side.

- a) 4 b) $\frac{5\sqrt{3}}{3}$ c) 5 d) $\frac{10\sqrt{3}}{3}$ e) $\frac{4\sqrt{15}}{3}$

- 25) Find the total surface area of a cylinder with a radius 4.6 mm and a height 9.2 mm. Round your answer to the tenth place.

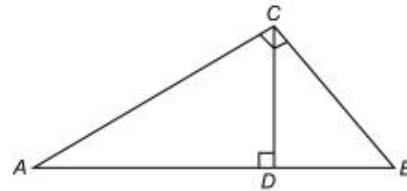
- a) 133.0 mm^2 b) 194.7 mm^2 c) 265.9 mm^2 d) 294.6 mm^2 e) 398.9 mm^2

- 26) An ice cream cone is 2.8 cm in radius and 12.7 cm deep. How many cubic centimeters of ice cream will it hold if it is filled and topped with a hemisphere with the same radius of the cone. State the volume to the nearest tenth.

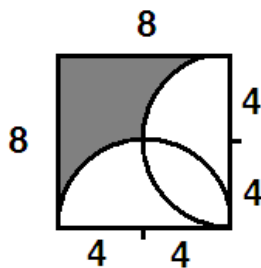
- a) 136.0 cm^3 b) 150.2 cm^3 c) 196.2 cm^3 d) 211.1 cm^3 e) 290.9 cm^3

- 27) Find one value for BD given the following: If $CD = 12 \text{ m}$ and $AB = 25 \text{ m}$

- a) 9 m b) 15 m
c) 20 m d) 24 m
e) 25 m



- 28) In the figure, find the area of the shaded region inside the square, but outside of the two semi-circles.

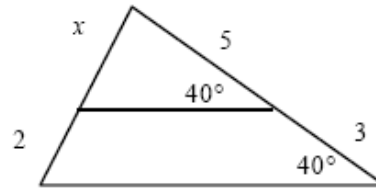


- a) 32 units^2 b) $16\pi + 32 \text{ units}^2$
c) $48 - 8\pi \text{ units}^2$ d) $64 - 4\pi \text{ units}^2$
e) $64 - 16\pi \text{ units}^2$

- 29) A lump of metal is submerged in a rectangular water tank 20 cm by 15 cm, raising the level of the water 0.35 cm. What is the volume of the metal?
- a) 150 cm^3 b) 135 cm^3 c) 125 cm^3 d) 105 cm^3 e) 95 cm^3
- 30) What is area of a circle that circumscribes a square which has an area of 25 centimeters squared?
- a) $50\pi \text{ cm}^2$ b) $25\pi \text{ cm}^2$ c) $5\pi\sqrt{2} \text{ cm}^2$ d) $\frac{25\pi\sqrt{2}}{2} \text{ cm}^2$ e) $\frac{25\pi}{2} \text{ cm}^2$

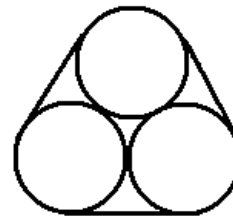
- 31) Find the length of x in the following diagram

- a) 3 units b) $\frac{10}{3}$ units
- c) $\frac{11}{3}$ units d) 4 units
- e) $\frac{13}{3}$ units



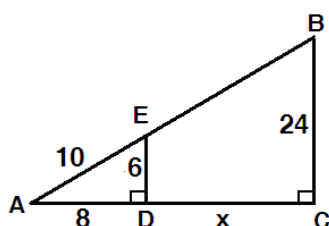
- 32) The perimeter of a right triangle is 30 meters and the sum of the legs is 17 meters. What is the area of this triangle?
- a) 30.0 m^2 b) 45 m^2 c) 55.5 m^2 d) 60 m^2 e) 144.5 m^2
- 33) The sides of a parallelogram 12 cm and 24 cm long, and one angle measures 125° . The area of the parallelogram is closest to
- a) 165 cm^2 b) 236 cm^2 c) 240 cm^2 d) 288 cm^2 e) 411 cm^2
- 34) Three wheels, each with a radius of 6, have their centers at respective vertices of a equilateral triangle. A belt is wrapped continuously around the wheels. Find the length of the belt.

- a) $18(2\pi + 1)$ b) $8\pi + 9$
- c) $6(\pi + 3)$ d) $12(\pi + 3)$
- e) $18(\pi + 1)$



- 35) A regular hexagon is inscribed in a circle. The perimeter of the hexagon is 36 units. Find the area of the circle.
- a) $24\pi \text{ units}^2$ b) $36\pi \text{ units}^2$ c) $48\pi \text{ units}^2$ d) $100\pi \text{ units}^2$ e) $144\pi \text{ units}^2$
- 36) Find the length of a chord which is the perpendicular bisector of the radius of length 16 in a circle.
- a) $9\sqrt{3}$ b) $16\sqrt{3}$ c) $8\sqrt{3}$ d) $16\sqrt{2}$ e) $8\sqrt{2}$

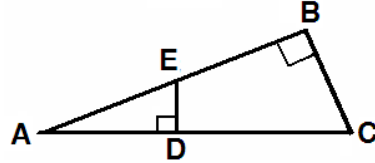
- 37) Given the right triangles ABC and AED. Find the length of x, when $\overline{ED} = 6$, $\overline{BC} = 24$ and $\overline{AD} = 8$.



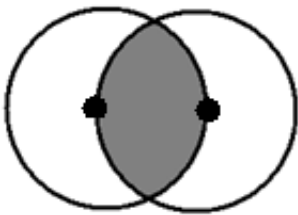
- a) 10 b) 24 c) $30\frac{2}{3}$
- d) 32 e) 40

- 38) The sides of a triangle are in arithmetic progression. The shortest side is 5 inches long and the perimeter of the triangle is 24 inches. The area of the triangle is closest to:
- a) 17.32 sq in b) 18.33 sq in c) 20.00 sq in d) 23.02 sq in e) 36.00 sq in
- 39) In the figure below, triangles ABC and ADE are right triangles with length of AB = 24, length of BC = 10, length of AC = 26, and length of AD = x. Find x to the nearest tenth so that the area of triangle ADE is 6 square units.

- a) 1.2 b) 2.2 c) 2.4
d) 5.4 e) 6.0



- 40) Find the length of a secant of the larger of two concentric circles with radii 10 cm and 6 cm. if the secant is tangent to the smaller circle.
- a) 8 cm b) 12 cm c) 16 cm d) 12π cm e) 16π cm
- 41) An arc of π inches in length is laid off on a circle of radius 5 inches. What is the measure of the central angle subtended by this arc?
- a) 48 degrees b) 42 degrees c) 36 degrees d) 25 degrees e) 18 degrees
- 42) Find the lateral area of a right prism that has an equilateral triangle for a base with side of 2.6 feet and has a prism height 3.2 feet.
- a) 8.32 sq ft b) 11.32 sq ft c) 12.48 sq ft d) 18.04 sq ft e) 24.96 sq ft
- 43) Let M and N denote the midpoints of two consecutive sides of a rectangle. The segment MN divides the area of the rectangle in the ratio
- a) 7 : 1 b) 4 : 1 c) 7 : 4 d) 4 : 3 e) 2 : 1
- 44) Find the area of the shaded region representing the intersection of two circles of radius 12, where each circle passes through the center of the other circle.



- a) $96\pi - 72\sqrt{3}$ b) $96\pi - 144\sqrt{2}$ c) $72\pi - 144\sqrt{2}$
d) $72\pi - 144\sqrt{3}$ e) $72\pi - 72\sqrt{3}$
- 45) Assuming that a cylindrical container can be mailed only if the sum of its height and the circumference do not exceed 270 centimeters, what are the dimensions of the cylinder with the largest volume that can be mailed?
- a) Height = 270 cm and Radius = $90/\pi$ cm b) Height = 180 cm and Radius = $90/\pi$ cm
c) Height = 90 cm and Radius = $90/\pi$ cm d) Height = 90 cm and Radius = $270/\pi$ cm
e) Height = 180 cm and Radius = 180/pi cm

- 46) When $x^5 - 6x^3 + 5x$ is factored completely with integer coefficients, then the sum of the factors is
- a) $x^2 + 3x - 5$ b) $x^2 + 2x - 5$ c) $x^2 + x - 7$ d) $x^3 + x + 5$ e) $2x^2 + x - 6$

- 47) A rock falls from a tower that is 336 feet high. As it is falling, its height is given by the formula $h = 336 - 16t^2$. Approximately how many seconds will it take for the rock to hit the ground to the nearest tenth?
- a) 2.5 sec. b) 4.6 sec. c) 18.3 sec. d) 17.9 sec e) 7056.0 sec.
- 48) A 50 year old man has two children. One of the children is 4 years older than the other. Eight years from now, the father will have the same age as the sum of the ages of the children. What is the age of the younger child now?
- a) 14 b) 16 c) 19 d) 24 e) 30
- 49) Write the quotient in standard form: $\frac{8 - 3i}{5 + 6i}$
- a) $\frac{-58}{11} - \frac{63}{11}i$ b) $-2 - \frac{63}{11}i$ c) $\frac{58}{61} - \frac{33}{61}i$ d) $\frac{22}{61} - \frac{63}{61}i$ e) $\frac{8}{5} - \frac{1}{2}i$
- 50) A bacterial culture has an initial population of 1000. If its population grows to 5000 in 2 hours, what will it be at the end of 4 hours? $P = Ie^{kt}$
- a) 1904 b) 25,000 c) 25,000,000 d) 30 e) 17
- 51) Solve completely for x: $3^{x^2} = 9^{x+4}$
- a) -4 b) -2 c) -4, 2 d) -2, 4 e) -4, -2
- 52) Identify one solution for the equation: $\log_4 x + \log_4 (x - 12) = 3$
- a) 16 b) 4 c) 12 d) 38 e) no real solution
- 53) License plates are made using 3 letters followed by 3 digits. How many plates can be made if repetition of letters and digits is allowed?
- a) 1,757,600 plates b) 308,915,776 plates c) 1,000,000 plates
d) 11,232,000 plates e) 17,576,000 plates
- 54) From a standard deck of 52 cards in how many ways can a hand of five cards be given in which there are no "face-cards"?
- a) 639,730 ways b) 658,008 ways c) 319,865 ways d) 201,376 ways e) 127, 946 ways
- 55) Find the distance from the point (4,3) to the line $y = -x + 3$
- a) 0 b) $2\sqrt{2}$ c) 2 d) $\sqrt{10}$ e) 4
- 56) A boat travels 24 km/hr on a still lake. On a river trip it takes 5 hours to go downstream with the current and 7 hours to make the return trip against the current. Find the rate of the current.
- a) 2 km/hr b) 3 km/hr c) 4 km/hr d) 5 km/hr e) 8 km/hr
- 57) Let $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{x+1}{x}$. Compute $(f \circ g)(x)$
- a) x b) 1 c) 2 d) $\frac{2x+1}{x}$ e) $\frac{x+1}{2x+1}$
- 58) State the domain of the function: $f(x) = \log_5(x^2 - 9x + 14)$
- a) All Real Numbers b) $\{x | x \neq 2, 7\}$ c) $\{x | 2 < x < 7\}$ d) $\{x | x > 0\}$ e) $\{x | x < 2 \text{ or } x > 7\}$

- 59) Find a pair of numbers A and B so that after performing the addition: $\frac{A}{x^2 - 1} + \frac{B}{x^2 + 3x + 2}$, the result reduces to: $\frac{6}{x^2 + x - 2}$
- a) A = 1 and B = 5 b) A = 2 and B = 4 c) A = 3 and B = 3
d) A = 4 and B = 2 e) A = 5 and B = 1
- 60) How many times will the graph of $|x| + |y| = 2$ intersect the graph of: $x^2 - y = 2$?
- a) 0 b) 1 c) 2 d) 3 e) 4
- 61) Frank flips two dimes and Terry flips 4 dimes. If the total number of Frank's heads is greater, then Frank wins, otherwise Terry wins. Find the probability that Frank wins.
- a) $\frac{25}{64}$ b) $\frac{1}{3}$ c) $\frac{7}{64}$ d) $\frac{11}{32}$ e) $\frac{21}{32}$
- 62) Solve the exponential equation (in terms of natural logs): $2^{x+7} = 6$
- a) $\ln(6) - \ln(2) - \ln(7)$ b) $\frac{\ln(2)}{\ln(6)} + \ln(7)$ c) $\frac{\ln(6)}{\ln(2)} + \ln(7)$
d) $\frac{\ln(6)}{\ln(2)} - 7$ e) $\frac{\ln(2)}{\ln(6)} - 7$
- 63) Solve the equation: $(e^x)^x \cdot e^{28} = e^{11x}$
- a) $\{-7, -4\}$ b) $\{7, -4\}$ c) $\{7\}$ d) $\{4\}$ e) $\{4, 7\}$
- 64) Give the equation of the horizontal asymptote, if any for the function: $R(x) = \frac{-3x^2}{x^2 + 6x - 55}$
- a) $y = 0$ b) $y = -55$ c) $y = -11$ and $y = 5$ d) $y = -3$ e) no horizontal asymptote
- 65) Square corners are cut from a rectangular piece of cardboard 16" by 20" and the remaining edges are folded up and secured to make an open box. The maximum volume of this box is closest to which of the following?
- a) 252 in^3 b) 320 in^3 c) 384 in^3 d) 420 in^3 e) 640 in^3
- 66) In the design of a swing set, the two metal posts that support the top bar each measure 8 feet. At the ground level, the posts are to be 6 feet apart. At what approximate angle should the two metal posts be secured to each other?
- a) 44 degrees b) 38 degrees c) 32 degrees d) 26 degrees e) 20 degrees
- 67) Find the acute angle between the lines: $3x - y = 10$ and $y = -2x + 4$
- a) 30° b) 37.5° c) 45° d) 52.5° e) 60°
- 68) A guy wire to the top of a tower makes an angle of 53 degrees with the level ground. At a point 32 feet farther from the guy wire ground base and in line with the base of the wire, the angle of elevation to the top of the tower is 25 degrees. What is the length of the guy wire?
- a) 54.55 ft b) 28.81 ft c) 60.47 ft d) 16.93 ft e) 35.55 ft
- 69) A box has the dimensions of 2 inches by 3 inches by 4 inches. Determine the approximate angle formed by the diagonal of the 2 X 3 side and the diagonal of the 3 X 4 side to the nearest degree.
- a) 45° b) 50° c) 60° d) 62° e) 65°

- 70) Complete the identity: $\sin(\alpha + \beta)\cos(\beta) - \cos(\alpha + \beta)\sin(\beta) = ?$
- a) $\sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$ b) $\sin(\alpha)\cos^2(\beta) - \sin(\alpha)\sin^2(\beta)$
c) $2\sin(\beta)\cos(\beta)(\sin(\alpha) - \cos(\alpha))$ d) $\sin(\alpha)$ e) $\sin(\alpha) - \cos(\alpha)$
- 71) A straight trail with a uniform inclination of 15 degrees leads from a lodge at an elevation of 600 feet to a mountain lake at an elevation of 6100 feet. What is the length of the trail to the nearest foot?
- a) 1,424 ft b) 5,694 ft c) 6,315 ft d) 21,250 ft e) 23,569 ft
- 72) Simplify: $\frac{\cot(x) - \tan(x)}{\cot(x) + \tan(x)}$
- a) $\cos(2x)$ b) $-\cos(2x)$ c) $1 - 2\cos^2(x)$ d) $\sin(2x)$ e) $\cot(2x)$
- 73) Find the open interval(s) where the function is decreasing: $f(x) = x^3 - 4x$
- a) $\left(\frac{2\sqrt{3}}{3}, \infty\right)$ b) $\left(-\infty, \frac{-2\sqrt{3}}{3}\right)$ c) $\left(\frac{-2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right)$ d) $\left(-\infty, \frac{2\sqrt{3}}{3}\right)$ e) $(-\infty, \infty)$
- 74) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\cos(x) - 1}{e^x - x - 1}\right)$
- a) 0 b) -1 c) 1 d) $-\infty$ e) ∞
- 75) The function: $f(x) = 8\pi x^2$ describes the volume, $f(x)$, of a right circular cylinder of height 8 feet and a radius "x" feet. If the radius is changing, find the instantaneous rate of the change of the volume with respect to the radius when the radius is 11 feet. Leave your answer in terms of pi.
- a) $16\pi \text{ ft}^3/\text{ft}$ b) $22\pi \text{ ft}^3/\text{ft}$ c) $88\pi \text{ ft}^3/\text{ft}$ d) $128\pi \text{ ft}^3/\text{ft}$ e) $176\pi \text{ ft}^3/\text{ft}$
- 76) Find the integral: $\int \ln(9x) dx$
- a) $x \ln(9x) + x + C$ b) $x \ln(9x) - x + C$ c) $x \ln(9x) - 9x + C$
d) $9x \ln(9x) - x + C$ e) $9x \ln(9x) + 9x + C$
- 77) Find the equation of the line tangent to the curve defined by: $x = t^3 + 5t - 1$ and $y = 3t^2 - 7t$ at the point corresponding to $t = 1$.
- a) $8x - y + 27 = 0$ b) $x + 8y + 27 = 0$ c) $8x + y - 27 = 0$
d) $x - 8y - 27 = 0$ e) $x + 8y - 27 = 0$
- 78) Find the minimum value of the function $f(x) = \frac{e^x}{x}$
- a) 1 b) e c) $\frac{1}{e}$ d) $\frac{1}{e^2}$ e) e^2
- 79) The value of the derivative at the point (2,1) for the curve $x^2 - xy - y^2 = 0$ is equal to:
- a) 2 b) $\frac{1}{2}$ c) 0 d) $\frac{3}{4}$ e) $\frac{4}{3}$
- 80) Find the area of the region enclosed by the curve $y^2 = 4 + x$ and the chord \overline{AB} joining two points of the curve, $A(-4,0)$ and $B(0,2)$.
- a) $\frac{52}{3}$ b) $\frac{32}{3}$ c) $\frac{16}{3}$ d) $\frac{4}{3}$ e) $\frac{1}{3}$

Here are the answers.

1.	D	2.	C	3.	A	4.	E
5.	C	6.	C	7.	A	8.	E
9.	D	10.	A	11.	D	12.	D
13.	C	14.	B	15.	E	16.	C
17.	C	18.	C	19.	B	20.	A
21.	C	22.	C	23.	A	24.	E
25.	E	26.	B	27.	A	28.	C
29.	D	30.	E	31.	B	32.	A
33.	B	34.	D	35.	B	36.	B
37.	B	38.	B	39.	D	40.	C
41.	C	42.	E	43.	A	44.	A
45.	C	46.	A	47.	B	48.	C
49.	D	50.	B	51.	D	52.	A
53.	E	54.	B	55.	B	56.	C
57.	E	58.	E	59.	D	60.	5
61.	C	62.	D	63.	E	64.	D
65.	D	66.	A	67.	C	68.	B
69.	C	70.	D	71.	D	72.	A
73.	C	74.	B	75.	E	76.	B
77.	B	78.	B	79.	D	80.	D

Question # 60 did not have an option that was correct – so problem was disqualified.