Divide and Conquer:
Digging Deeper into Division

Barbara Boschmans
Barbara.Boschmans@nau.edu

Northern Arizona University, Flagstaff, AZ

Students' Confusion with Division

Some students were having trouble interpreting calculator decimal answers when dividing whole numbers.

For Example:

47 ÷ 6 = 7.833333333…

Some of their answer would be 7 with a remainder of 8.

They never gave it a second thought that the remainder was greater than the divisor…

Even after showing the Division Algorithm: Given any two integers a and b where $b>0$, there exists a unique pair of integers r and q such that

$a \div b = q \text{ R } r$ if $a = b \times q + r$

and $0 \leq r < |b|$

students still didn't seem to have a "gut-feeling" of what division meant.

So, introducing them to the division models was the next step in building "division sense" aka "gut-feeling".

CCSS.Math.Grade 3 Introduction

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; …, and division is finding an unknown factor in these situations. **For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size**.

CCSS.Math.Grades 5-6 Introduction

Gr 5: Instructional time should focus on developing fluency with addition and subtraction of fractions, and **developing understanding of the** multiplication of fractions and of **division of fractions** in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions). Gr 6: Instructional time should focus on completing **understanding of division of fractions**.

Models for Division

1. Division as Repeated Subtraction = Measurement Division. For equal-sized group situations, division can require **finding the unknown number of groups**.

"How many groups?"

2. Division as Partition = Division by Sharing. For equal-sized group situations, division can require **finding the unknown group size**.

"How many in each group?"

3. Division as a Missing Factor.

Whole Number Division as Repeated Subtraction

Let's use the original example: 47 ÷ 6

The question we can ask is:

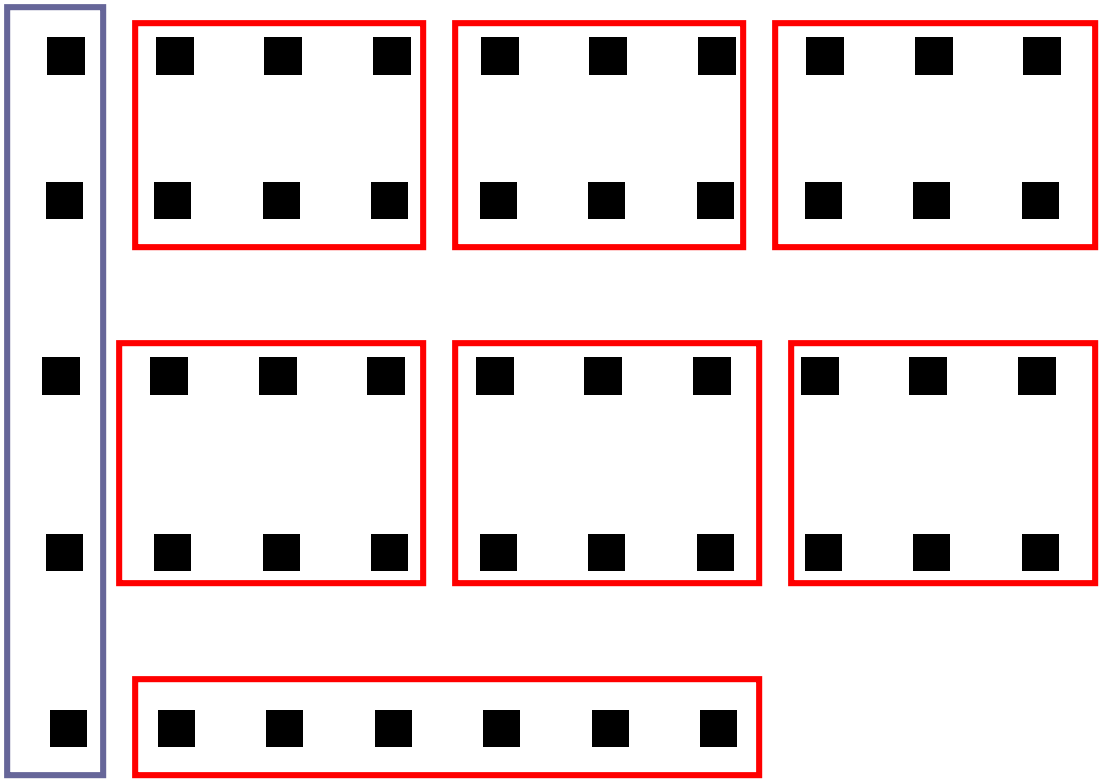
How many groups of 6 are there in 47?

[For equal-sized group situations, division can require finding the unknown number of groups]

or: How many times can we subtract 6 from 47?

47 – 6 = 41
41 – 6 = 35
35 – 6 = 29
29 – 6 = 23
23 – 6 = 17
17 – 6 = 11
11 – 6 = 5

We can subtract 6 from 47 **seven** times with 5 remaining, giving us the answer
47 ÷ 6 = **7** R 5



In 47 there are 7 groups of 6 and 5 leftover

Whole Number Division as Partition

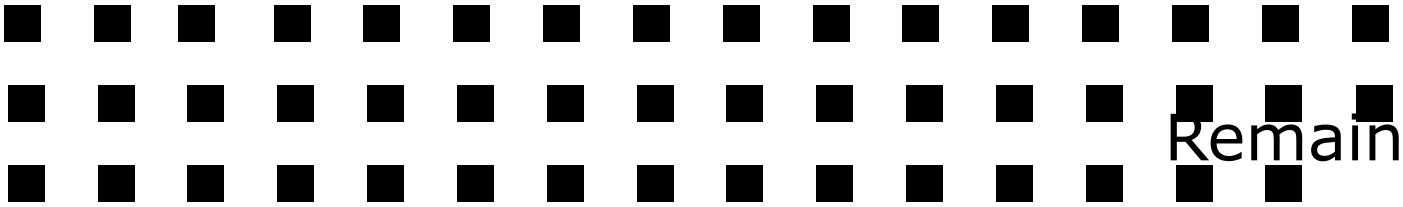
Let's use the original example again: 47 ÷ 6

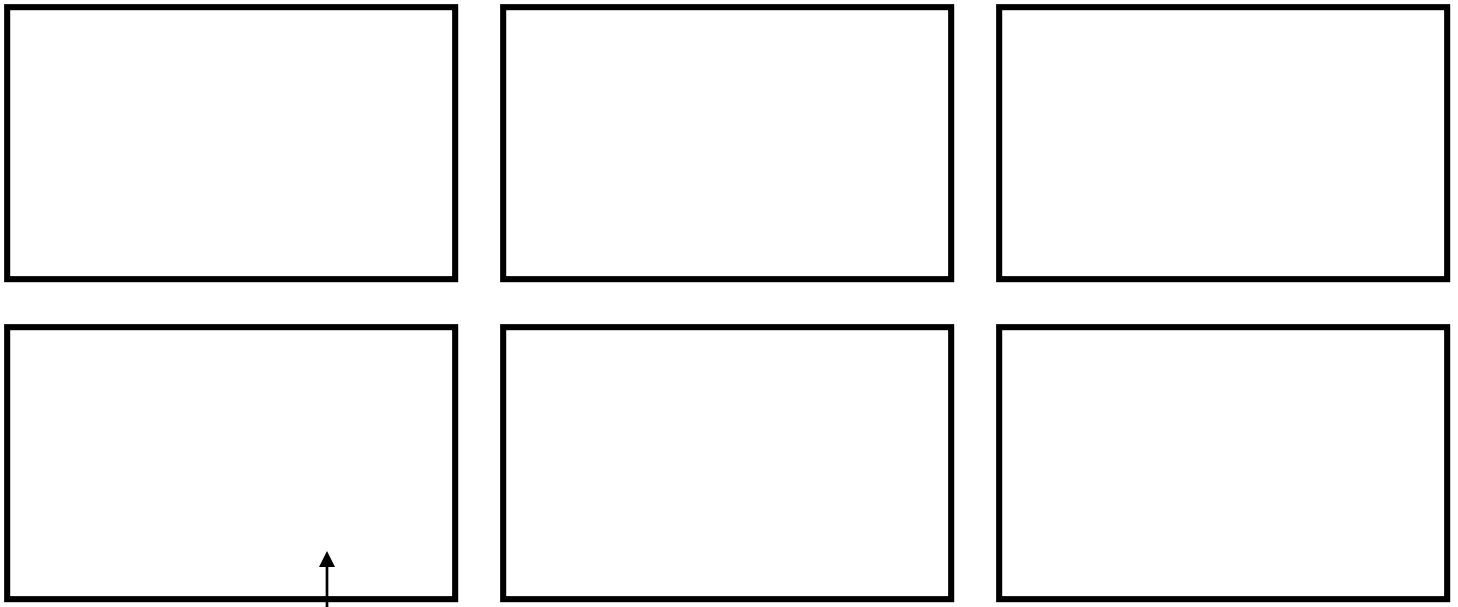
The question we need to ask is:

How can I divide (partition/share) 47 to 6 groups?

[For equal-sized group situations, division can require finding the unknown group size.]

A good analogy for Division as Partition is dealing cards: you deal one by one. So we want to distribute 47 cards to 6 people.





47 ÷ 6 = 7 R 5

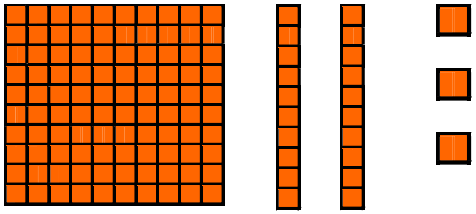
Division of Whole Numbers

Using base ten blocks we can illustrate division with both models. Depending on the divisor, one model of division can be a more efficient way of solving a problem than another model.

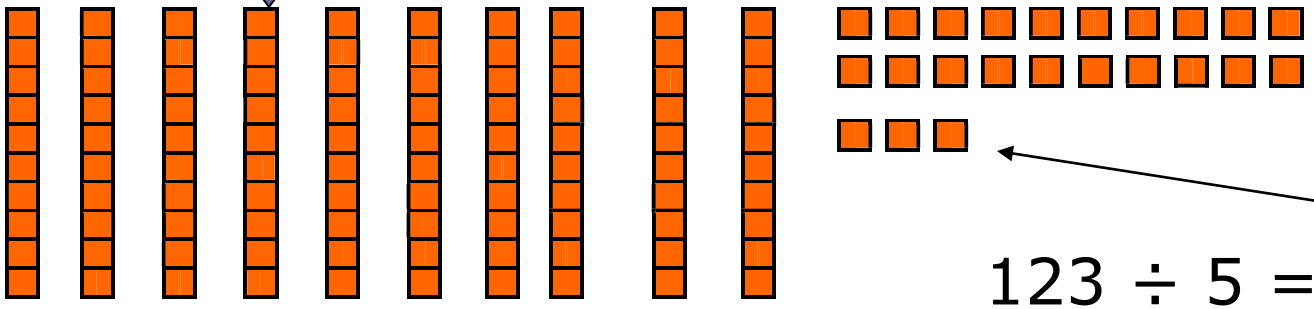
123 ÷ 5 would be better solved with a partition model than a repeated subtraction model. WHY?

123 ÷ 12 would be better solved with a repeated subtraction model than a partition model. WHY?

123 ÷ 5

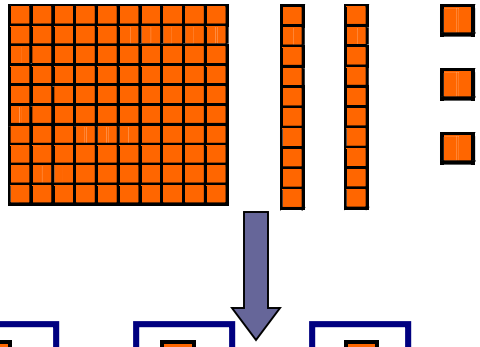


In order to divide 123 into 5 groups we trade one hundred for ten tens and two tens for twenty units.

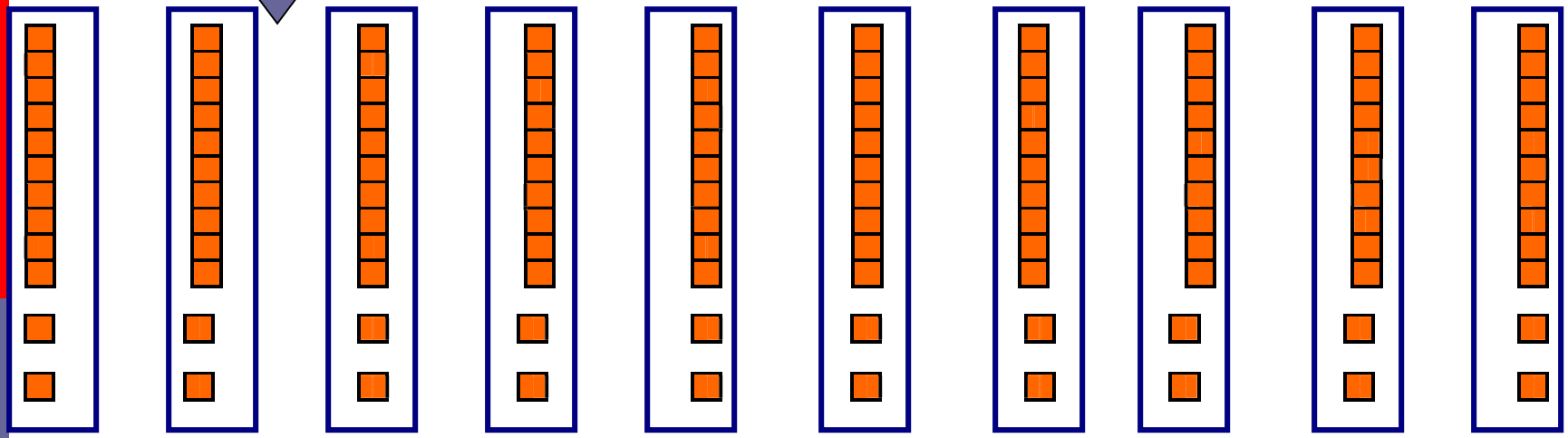


123 ÷ 5 = 24 R 3

123 ÷ 12

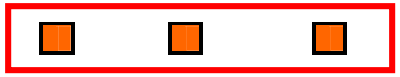


In order to subtract groups of 12 from 123 we trade one hundred for ten tens and two tens for twenty units.



There are 10 groups of 12

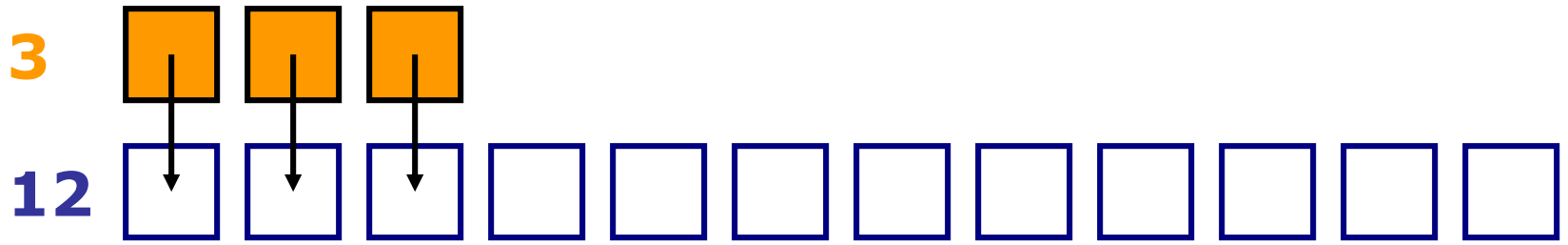
with 3 leftover



So where is the decimal?

On a calculator, 123 ÷ 12 = 10.25

The decimal part (.25) is the part of 12 that also fits into 123. Let's think about this, when 12 is repeatedly subtracted from 123, there are 10 groups of 12 with 3 left over. If we think about what part of 12 fits into 3, we create the decimal (.25).

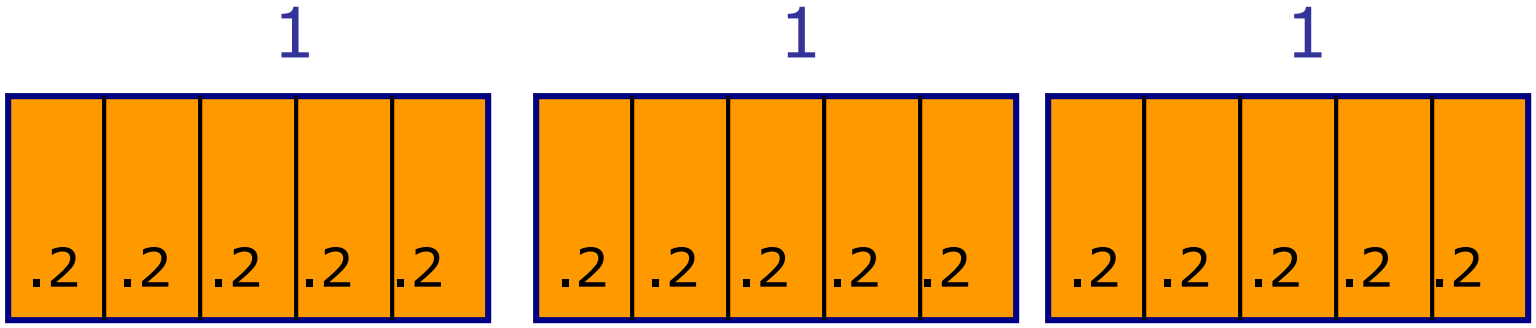


This part of 12 fits into 3 =3/12 = 1/4 = .25 !

So where is the decimal?

On a calculator, 123 ÷ 5 = 24.6

The decimal part (.6) is 3 divided into 5 groups. Let's think about this, when 123 is divided into 5 groups, there are 24 in each group with 3 left over.If we divide the 3 into 5 equal parts and distribute them equally to the 5 groups, each group receives .2 + .2 + .2 so we create the decimal (.6).



Your Turn!

Our original problem:

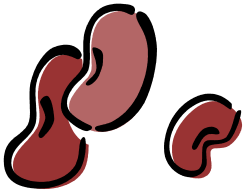
On a calculator, 47 ÷ 6 = 7.833333….

Let's think about this, when 47 is divided into 6 groups, there are 7 in each group with 5 left over. 47 ÷ 6 = 7 R 5.

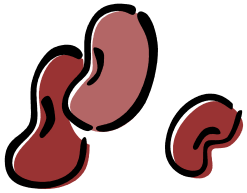
If we divide the 5 into 6 equal parts and distribute them equally to the 6 groups, each group receives:

$$0.1\overline{6} + 0.1\overline{6} + 0.1\overline{6} + 0.1\overline{6} + 0.1\overline{6} = 0.8\overline{3}$$

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6} = 0.8\overline{3}$$



Game Time!



- Shuffle your cards numbered 6-18. Put the cards facedown.
- Player A takes a card and takes the number of beans shown on the card. Then Player A roles the die. The number on the die is the number of groups you need to make using your beans. Player A's score is the number of beans in each group. If there is no remainder, Player A's score is double the number of beans in one group.
- Next Player B takes a card, beans and rolls the die following the same rules from Player A.
- Keeping track of scores, the winner is the highest scorer after five rounds!

From Everyday Mathematics Games

Interpreting the Remainder

Divide and decide what to do with the remainder: keep it as a whole number, or keeping your answer in fraction/decimal form. Sometimes you'll need to round your answer up!

1. Isabelle is having a birthday party and a bunch of balloons costs $6, how many bunches can she buy with $75?
2. I have a very long rope of licorice: 93 inches long! If I cut it into 12 pieces, how long is each piece?

Interpreting the Remainder (continued)

3. It costs $50 to admit 8 people to the zoo. What is the cost per ticket?
4. Noelle has 75 DVDs. If a basket can store 9 DVDs, how many does she need to store all her DVDs?
5. Juliette has 348 pennies. If she wants to divide them into sets of 16, how many sets can she make? How many pennies are left over?

Game Time!

Take out your calculator, pair up and play!

Player A chooses a secret number between 1 and 100.

Player B guesses the number.

Player A enters the guess and divides it by the secret number.

Player A reads the answer to Player B.

Player B guesses until the secret number is found.

Level 1: On a TI-15 use the "Int ÷" key to read the answer in whole number remainder form.

Level 2: On a TI-15 use the "F ↔D" key to read the answer in fraction form.

Level 3: Read the answer in decimal form.

From Everyday Mathematics Games

Division of Fractions

Three Cases:

- Division of a fraction by a whole number

  [For equal-sized group situations, division can require finding the unknown group size.]

- Division of a whole number by a fraction

  [For equal-sized group situations, division can require finding the unknown number of groups]

- Division of a fraction by a fraction

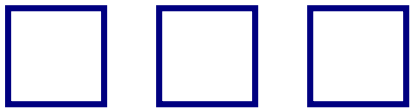
  [For equal-sized group situations, division can require finding the unknown number of groups]

Division of a Whole Number by a Whole Number

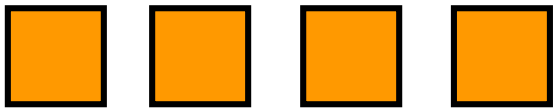
Divide 3 ÷ 4

"How many groups of 4 are there in 3?"

3:



4:



Since 4>3, only a part of 4 will fit into 3.
What part fits?
3 out of 4 or $\frac{3}{4}$

Division of a Fraction by a Whole Number

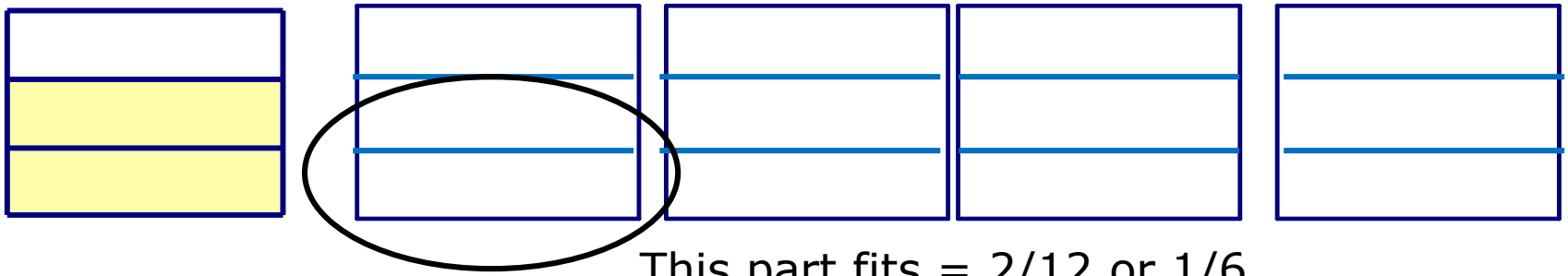
We can use a rectangle (area) model to divide fractions. We will uses the measurement model of division.

Divide 2/3 ÷4.

"How many groups of 4 are in 2/3?"

Shade 2/3 of the whole: Then find 4 wholes:

Now we divide the 4 wholes into thirds and since 4>2/3, we need to find out what part of 4 fits into 2/3.



This part fits = 2/12 or 1/6

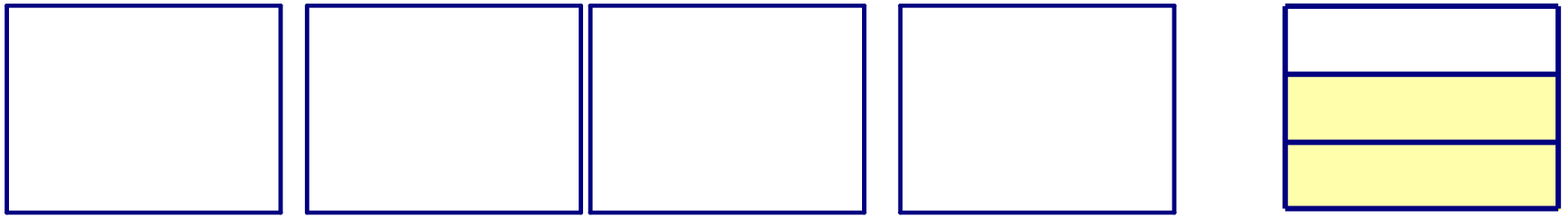
Division of a Whole Number by a Fraction

We can use a rectangle (area) model to divide fractions. We will uses the measurement model of division.

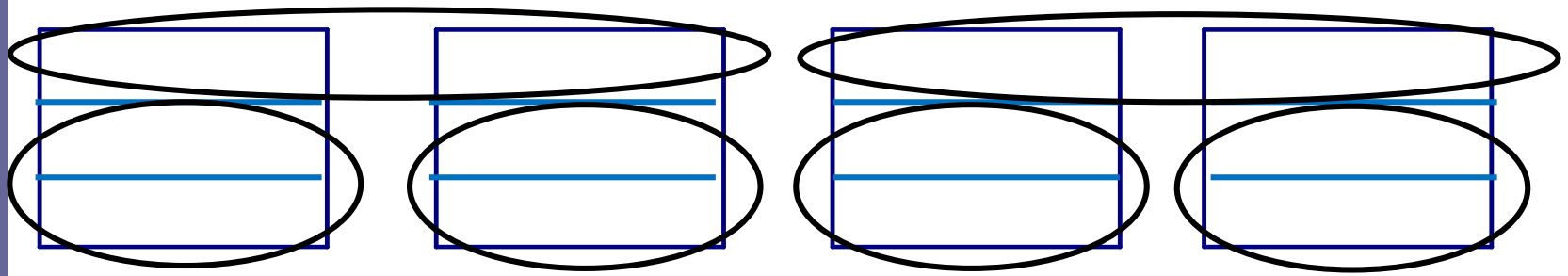
Divide 4 ÷2/3.

"How many groups of 2/3s are in 4?"

Shade 4 of the whole: Then find 2/3 of the whole:



Now we divide the 4 into thirds horizontally and find how many groups of 2/3s are in 4.



There are six groups of 2/3 in 4.

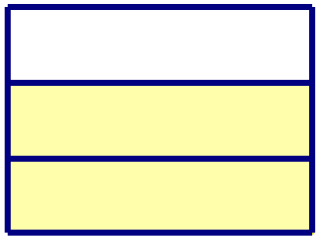
Division of a Fraction by a Fraction

We can use a rectangle (area) model to divide fractions. We will uses the measurement model of division.

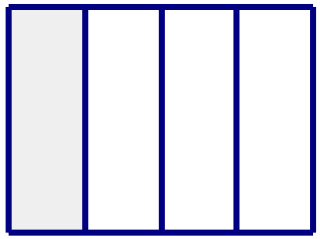
Divide 2/3 ÷1/4.

"How many groups of 1/4s are in 2/3?"

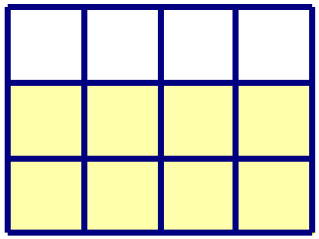
Shade 2/3 of the whole:

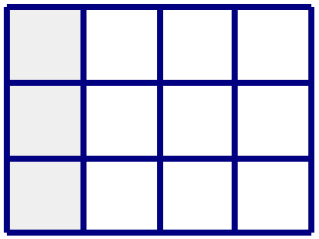


Then find 1/4 of the whole:



Now we divide the thirds into fourths vertically and the fourths into thirds horizontally. Note that 2/3 is equivalent to 8/12 and 1/4 is equivalent to 3/12.





Let's give this a context:

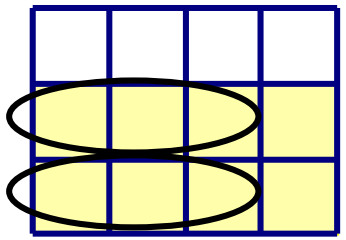
I have 2/3 lb of ground beef and I want to make quarter pounders (1/4 lb of ground beef).

A. How many quarter pounders can we make? How much of a lb of ground beef is left over?
B. How many quarter pounders can we make? If we want to use all of our meat, what fraction of a quarter pounder can we make with the leftover ground beef?

Division of Fractions (continued)

A. One 1/4 fits:

A second 1/4 fits:

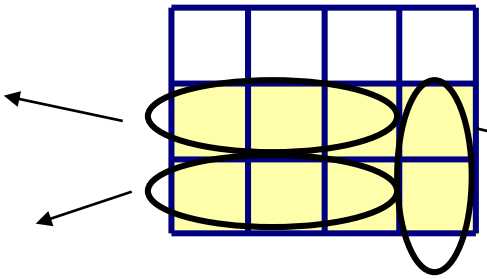


2/3 ÷ 1/4 = 2 R 2/12 = 2 R 1/6

So we can make 2 quarter pounders and we have 1/6lbs of ground beef leftover.

B. One 1/4 fits:

A second 1/4 fits:



:and 2/3 of 1/4 fits

2/3 ÷ 1/4 = 2 2/3

So we can make 2 quarter pounders and another 2/3 of a quarter pounder.

Questions/Comments

Thank you for participating!

Barbara Boschmans
Barbara.Boschmans@nau.edu